lend themselves to easy control in the laboratory and 2) the feasibility of a real transformation consistent with the latter constraints is difficult to imagine. Yet, these arguments become untenable when subjected to deeper scrutiny because all constraints have a mathematically virtual nature and are associated exclusively with the final state of equilibrium, rather than with the transformation that the system actually follows to reach that state. Such an interpretation is the only one consistent with Eqs. (13) and (14) and Eqs. (17) and (18) being equivalent and leading to the same equilibrium conditions.

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Thermal Effects of Particles on Hypersonic Ablation

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Nomenclature

a	=	particle radius
a_1	=	particle temperature profile parameter
D	=	particle drag in the melt layer
H_1, H_2	=	functions of $\tilde{h}$, Eqs. (19a) and (19b)
h	=	average heat convection coefficient
$\tilde{h}$	=	dimensionless convection coefficient, Eq. (13a), Biot number
Q	=	total heat loss from the particle to the melt
$\dot{q}$	=	rate of heat loss from the particle to the melt
r	=	radial distance from the particle center
$\tilde{r}$	=	dimensionless radial distance, r/a
T	=	temperature
t	=	time
$\tilde{t}$	=	dimensionless time, t/t_f
$\tilde{u}$	=	dimensionless temperature inside the particle, Eq. (13b)
V	=	particle velocity in the melt layer
$\tilde{V}$	=	dimensionless particle velocity, $V/(z^*/t_f)$
z	=	distance from the melt–solid interface
$\bar{z}$	=	distance from the air–melt interface, $z^* - z$
z^*	=	melt-layer thickness
α_p	=	particle thermal diffusivity
$\tilde{\alpha}_p$	=	dimensionless particle thermal diffusivity, Eq. (13a)
η_2	=	z/z^*
λ_a	=	$(\rho_2/\rho_p)(z^*/a)$
$\bar{\nu}_2$	=	average kinematic viscosity of the melt
ρ	=	density

Subscripts

f	=	final
m	=	conditions on the ablation surface
p	=	particle
2	=	conditions in the melt layer
∞	=	freestream conditions

Superscript

$*$	=	conditions at the air–melt interface

I. Introduction

IT is well known (for example, see Ungar[1]) that thermal protection systems used in severe aerodynamic heating environments associated with hypersonic flights are often subject to the effects of particles, either in solid form or in the form of liquid droplets, or both, in the gas stream. These particles carry large thermal and kinetic energies as they enter the thermal protection system, and their effects on the performance of the system are, thus, expected to be significant.

In this Note, we present a simplified model to assess the thermal effects of these particles on aerodynamic ablation near the stagnation point of a circular-nosed body in hypersonic flight. Only the case of sparse particles of small size is considered, and the particles will be considered as rigid spheres. These assumptions allow considerable simplifications to be made in the analysis. Some preliminary results are also presented, and their possible refinements are discussed. The dynamic effects that could contribute significantly to the mechanical erosion of the ablative materials will require a separate model for material responses, and they will not be considered here.

The study will be based on the earlier work by Zien and Wei[2,3] on the modeling of particle-free hypersonic ablation. The central part of the model is a thin melt layer formed by the molten ablative material (see also Lees[4] and Hidalgo,[5] among others), which is coupled to the airflow on the one side and to the ablating solid on the other (Fig. 1). The melt layer plays a critical role in providing a heat shield to the aerodynamic body through conduction and convection currents in the layer. Because we consider only the case of sparse particles of small size (compared to the dimension of the melt layer), the interaction between particles and the effects of particles on the basic ablation field can be neglected in the first approximation.

To fix the idea, we consider the case where the particle enters the melt layer at a given (high) velocity, and it is in thermal equilibrium with the hot carrier gas on entry. The particle travels through the nonuniform but known temperature field of the melt layer as given in Refs. 2 and 3, and its motion is inertia dominated. The temperature field inside the spherical particle is assumed to be one dimensional (in the radial direction) and time dependent, and the heat loss to the surrounding melt is expressed approximately by a constant average heat transfer coefficient.

The simplified model is, thus, amenable to analytical treatments. Relevant dimensionless parameters for the problem are also identified, and they are used in the presentation of results. Possible refinements of these preliminary results are discussed. It is hoped that the results of the present study will be useful as a starting point for the study of dynamic effects of particles on ablation, for which additional models for material response will need to be incorporated

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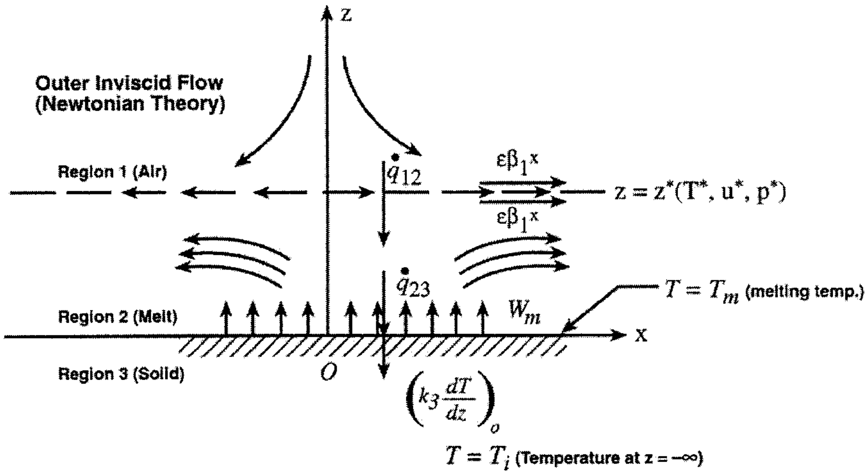

Fig. 1 Hypersonic ablation model structure (coordinate system fixed on ablation front).

(for example, see Cheung et al.[6]). Many details are omitted here but appear in Ref. 7.

II. Model and Analysis

A. Particle Dynamics

Consider a spherical particle of radius a entering the melt layer of the basic particle-free ablation model proposed by Zien and Wei.[2,3] The particle has an initial velocity V_i, which, in most cases, can be taken as approximately equal to the freestream velocity, as suggested by Ungar.[1] Because the melt flow velocity in the basic melt layer is an order of magnitude smaller than the air velocity in the boundary layer,[2,3] it can be neglected in the first approximation of particle dynamics. Let the melt-layer thickness be z^*. (We will follow the nomenclature of Refs. 2 and 3.) For simplicity, we consider the case of vertical entry, although the generalization to oblique entry should cause little difficulty. We will use the coordinate $\bar{z}$ to measure the distance traveled by the particle from its entry at the air–melt interface. Thus, we have

$$\bar{z} = z^* - z \tag{1}$$

where $z = 0$ represents the melt–solid interface in the original ablation mode (Fig. 1).

The dynamics of the particle that travels at a very high velocity through the melt layer will be inertia dominated. The Reynolds number based on the particle radius and some typical reentry conditions at Mach 16 with the air–quartz ablation system[5] is estimated to be on the order of 10^2 (Ref. 7). We note that, for a less viscous ablative material, the corresponding Reynolds number will be even greater.

For inertia-dominated flows, the drag on the particle, D, can be estimated by the following formula based on the Newtonian impact theory (see Ungar[1] and Anderson[8]):

$$D = \pi a^2 \rho_2 V^2 \big/ 2 \tag{2}$$

where ρ_2 is the density of the melt and V is the instantaneous particle velocity in the $\bar{z}$ direction. In this approximation, the viscous drag is neglected compared to the inertia drag. This is a crude approximation, and it is used here mainly to obtain a relatively simple expression for the particle trajectory that can be effectively employed in the heat transfer calculations later. With this drag approximation, we can easily integrate the equation of motion of the particle to obtain the following particle velocity:

$$V/V_i = \exp\left[-\tfrac{3}{8}(\rho_2 \bar{z}/\rho_p a)\right] \tag{3}$$

where V_i is the entry velocity of the particle.

The terminal (impact) velocity of the particle, V_f, is, thus, obtained as

$$V_f/V_i = \exp\left(-\tfrac{3}{8}\lambda_a\right) \tag{4}$$

where we have introduced an important dimensionless parameter λ_a, the deceleration parameter, defined as

$$\lambda_a = \left(\rho_2 z^*\right)\big/(\rho_p a) \tag{5}$$

It will become obvious that, in this simplified model, this parameter alone determines the dynamics of the particle in the melt layer. Note that, in general, the impact velocity is finite.

Integrating Eq. (3) with respect to t gives the particle trajectory $\bar{z}(t)$, with the initial condition, $\bar{z}(0) = 0$, as follows:

$$\bar{z}/z^* = \tfrac{8}{3}(1/\lambda_a)\ln\left[1 + \tfrac{3}{8}(\rho_2/\rho_p)(V_i t/a)\right] \tag{6}$$

The total time of the particle travel, t_f, is the time at which $\bar{z} = z^*$, and it is easily found from Eq. (8) as

$$\left(V_i\big/z^*\right)t_f = \tfrac{8}{3}(1/\lambda_a)\left[\exp\left(\tfrac{3}{8}\lambda_a\right) - 1\right] \tag{7}$$

It is convenient to use t_f as the timescale and to introduce the dimensionless time $\tilde{t}$,

$$\tilde{t} = t/t_f \tag{8}$$

in the following discussion. Thus, we write Eq. (6) as

$$\bar{z}/z^* = \tfrac{8}{3}(1/\lambda_a)\ln\left\{1 + \left[\exp\left(\tfrac{3}{8}\lambda_a\right) - 1\right]\tilde{t}\right\} \tag{9}$$

The particle trajectory as given by Eq. (9) is shown in Fig. 2 with λ_a as a parameter. Note that in the limit of $\lambda_a \to 0$, which corresponds to the case of very small density ratio ρ_2/ρ_p and, hence, a vanishing drag, Eq. (9) gives $\bar{z}/z^* \to \tilde{t}$. This means that the particle travels at a constant velocity equal to its initial velocity V_i as expected for the case of zero resistance ($\lambda_a = 0$).

B. Particle Heat Transfer

1. Model

The particle passes through a nonuniform temperature field in the melt layer across which a considerable temperature drop ($T^* - T_m$) occurs[2,3] (Fig. 1). Heat transfer from the hot particle to the relatively cold surrounding melt will take place, and it amounts to an external heating to the melt field and, hence, to the thermal protection system. This heating represents the thermal effects of the particles.

To calculate the heat transfer, we will make the following simplifying assumptions in the present preliminary study. We assume that the particle has a uniform temperature at the entry to the melt layer equal to the temperature of the air at the air–melt interface T^*. We

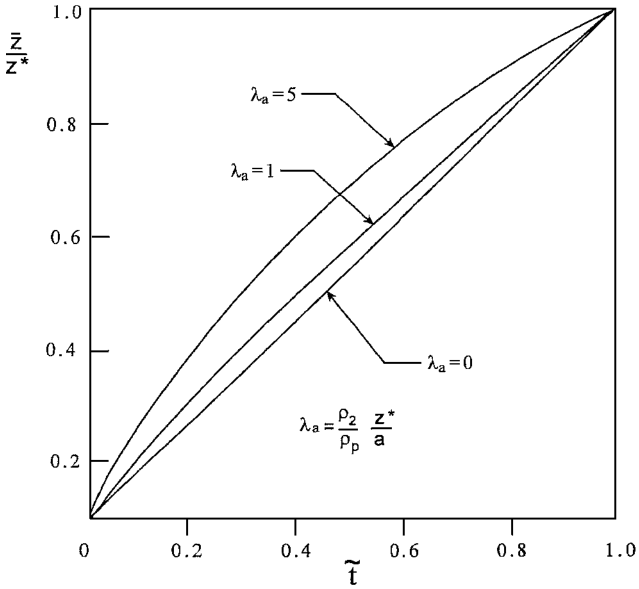

Fig. 2 Particle trajectory.

will also assume that the subsequent temperature variation inside the particle can be approximated by a one-dimensional transient heat conduction model with constant (average) thermal conductivity k_p. The heat loss to the surrounding melt whose temperature decreases as seen by the traveling particle can be represented by an average convection coefficient h. Of course, the determination of h is by no means an easy task, but it is assumed as given in the present analysis. At any instant in time, the ambient temperature $T_2(z)$ that the moving particle experiences varies around the particle surface, but for simplicity, we will take the temperature to be that of the melt field at the location of the center of the particle. This will become a better approximation as the particle size becomes smaller compared to the melt-layer thickness.

In terms of the spherical coordinates fixed at the center of the particle, the equation of heat conduction inside the particle and the appropriate initial and boundary conditions are as follows:

$$\frac{\partial T_p}{\partial t} = \alpha_p \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T_p}{\partial r}\right) \tag{10a}$$

$$t = 0\colon T_p = T^* \quad \text{(const) for all r} \tag{10b}$$

$$r = 0\colon \frac{\partial T_p}{\partial r} = 0 \quad \text{(symmetry)} \tag{10c}$$

$$r = a\colon -k\frac{\partial T_p}{\partial r} = h[T_p - T_2(z)] \tag{10d}$$

In the preceding system, $T_p(r, t)$ is the particle temperature distribution, and h is the average heat transfer coefficient taken to be constant. $T_2(z)$ is the temperature of the melt layer of the original particle-free ablation model as obtained in Refs. 2 and 3, and z is related to t through the particle trajectory, Eqs. (1) and (9).

Note that the rate of heat loss to the melt layer, $\dot{q}$, is calculated as follows:

$$\dot{q} = 4\pi a^2\left(-k_p\frac{\partial T_p}{\partial r}\right)_{r=a} = 4\pi a^2 h\{(T_p)_{r=a} - T_2[z(t)]\} \tag{11}$$

and the total heat loss to the melt layer by the particle during its entire trajectory, Q, is simply

$$Q = \int_0^{t_f} \dot{q}\,\mathrm{d}t \tag{12}$$

It is convenient to use dimensionless quantities in the solution of the preceding system of equations. Therefore we introduce the following dimensionless variables:

$$\tilde{t} = t/t_f, \qquad \tilde{r} = r/a, \qquad \tilde{\alpha}_p = \alpha_p t_f / a^2$$

$$\tilde{h} = ha/k_p, \qquad \tilde{V} = V/(z^*/t_f) \tag{13a}$$

and

$$\tilde{u} = \tilde{r}\frac{T_p - T_2(t)}{T^* - T_m}, \qquad G_2(\eta_2) = \frac{T_2 - T_m}{T^* - T_m}$$

$$\tilde{Q} = \frac{Q}{4\pi a k_p (T^* - T_m) t_f} \tag{13b}$$

Note that $\tilde{h}$ in Eq. (13a) is known as the Biot number and G_2 in Eq. (13b) is the temperature distribution in the melt layer of the basic ablation model.[2,3] Also, we have

$$\eta_2 = z/z^* = 1 - \bar{z}/z^* = \eta_2(\tilde{t}) \tag{14}$$

where the particle trajectory, Eq. (9), is used.

The dimensionless form of the heat conduction equations becomes

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} = \tilde{\alpha}_p \frac{\partial^2 \tilde{u}}{\partial \tilde{r}^2} + \tilde{V}\tilde{r}\frac{\mathrm{d}G_2}{\mathrm{d}\eta_2} \tag{15a}$$

$$\tilde{u}(\tilde{r}, 0) = 0 \tag{15b}$$

$$\tilde{u}(0, \tilde{t}) = 0 \tag{15c}$$

$$-\left(\frac{\partial \tilde{u}}{\partial \tilde{r}}\right)_{\tilde{r}=1} = (\tilde{h} - 1)(\tilde{u})_{\tilde{r}=1} \tag{15d}$$

The additional term on the right-hand side of Eq. (15a) is positive, and it represents the increase of the temperature difference $(T_p - T_2)$ with time, caused by the motion of the particle in the direction of decreasing T_2.

2. Solutions and Discussion

Solutions of Eqs. (15a–15d) can be easily obtained in simple closed forms by an integral method as follows. We first assume a temperature profile that satisfies all of the boundary conditions and the initial condition given in Eqs. (15b–15d). As a first attempt, we will choose the following simple polynomial form:

$$\tilde{u} = a_1(\tilde{t})\{1 - [\tilde{h}\tilde{r}/(1 + \tilde{h})]\}\tilde{r} \tag{16}$$

where $a_1(\tilde{t})$ is the profile parameter [with $a_1(0) = 0$] that will be determined by requiring that the approximate profile $\tilde{u}$ in Eq. (16) satisfy Eq. (15a) in its integrated form. We note that the choice of the approximate profile in the integral method is by no means unique. In fact, a profile of polynomials with a higher degree is perhaps more appropriate because it will ensure the original symmetry condition $(\partial T_p/\partial r)_{r\to 0} \to 0$ be satisfied. Interested readers are referred to Ref. 7 for a discussion of this point.

Integrating Eq. (15a) across the radius of the particle, we get the following ordinary differential equation:

$$\frac{\mathrm{d}a_1}{\mathrm{d}\tilde{t}} + H_1\tilde{\alpha}_p a_1 = H_2\tilde{V}G_2'(\eta_2) \tag{17}$$

with

$$a_1(0) = 0 \tag{18}$$

The constants H_1 and H_2 in Eq. (20) are functions of the Biot number defined as

$$H_1 = 4\tilde{h}/(1 + \tilde{h}/3) \tag{19a}$$

$$H_2 = (1 + \tilde{h})/(1 + \tilde{h}/3) \tag{19b}$$

The solution for $a_1(\tilde{t})$ is readily obtained analytically[7] in terms of the parameters H_1, H_2, G_2, λ_a, and $\tilde{\alpha}_p$, and it can be expressed symbolically as

$$a_1(\tilde{t})/H_2 = f(\tilde{t};\, H_1\tilde{\alpha}_p;\, \lambda_a) \tag{20}$$

for a given melt-layer temperature field G_2. The function f is given explicitly in Ref. 7. Several important quantities related to the heat transfer problem can now be studied on the basis of the approximate solution of $\tilde{u}$.

The total heat released by the particle during its passage through the melt layer is given by

$$\tilde{Q} = \tilde{h}\int_0^1 (\tilde{u})_{\tilde{r}=1}\,\mathrm{d}\tilde{t} = \frac{\tilde{h}}{1+\tilde{h}}\int_0^1 a_1(\tilde{t})\,\mathrm{d}\tilde{t} \tag{21}$$

The integral in Eq. (21) can be conveniently expressed in terms of a_1 (1) by integrating Eq. (17) over the time interval to give the result of nondimensional heat loss as

$$\tilde{Q}\tilde{\alpha}_p = \tfrac{1}{4}\{1 - [a_1(1)/H_2]\} \tag{22}$$

Equation (22) can be conveniently written as

$$\tilde{Q}\tilde{\alpha}_p = f(1;\, H_1\tilde{\alpha}_p,\, \lambda_a) \tag{23}$$

in view of Eq. (20).

The temperature difference between the center of the particle and its surface as a function of time can also be obtained in term of $a_1(\tilde{t})$ (Zien[7]).

For the purposes of demonstrating the results of these computations, we have chosen a particular melt-layer temperature field $G_2(\eta_2)$, which is taken from Ref. 2.

The results of $\tilde{Q}\tilde{\alpha}_p$ are shown in Fig. 3 as a function of $H_1\tilde{\alpha}_p$ for $\lambda_a = 1$ and $\lambda_a = 5$. The results show that for a given value of the resistance parameter λ_a, the total heat loss by the particle increases with increasing values of the Biot number for the same $\tilde{\alpha}_p$. Also, the total heat loss increases with increasing values of the resistance parameter (hence, longer travel time) for the same Biot number and $\tilde{\alpha}_p$. These results all seem to be intuitively correct. Some results of $a_1(\tilde{t})/H_2$, a measure of the temperature difference across the particle radius, are available in Ref. 7.

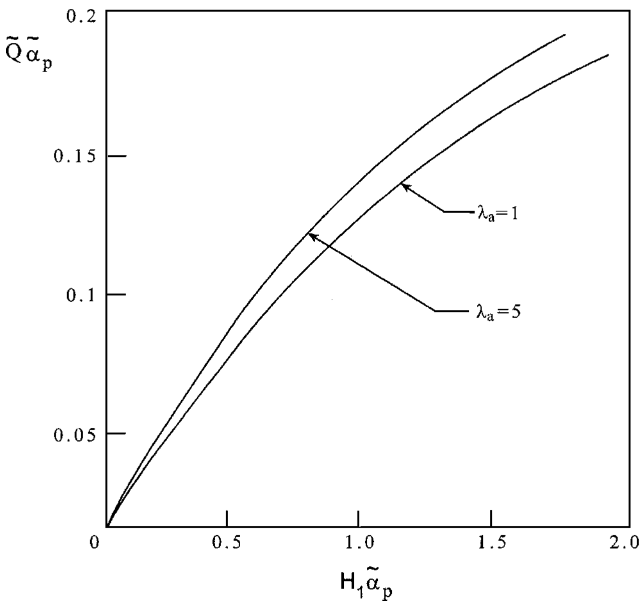

Fig. 3 Total heat loss from particle, basic ablation model Ref. (2): $(\varepsilon, R^*, m, N_m, Pr_1, Pr_2, \bar{\nu}_2/\nu_1^*)$ = (0.2, 0.4, 0.1, 0.75 × 10^{-6}, 0.7, 5.0, 0.25 × 10^{-3}).

III. Conclusions

We have presented a model for the study of thermal effects of solid particles carried by the gas stream on thermal ablation in hypersonic flow. A basic assumption and, hence, a limitation of this simplified model is that of sparse particles, and it is made so that the interactions between particles and the effects of particles on the gas stream and on the basic particle-free ablation model can be neglected in the first order of the approximation. The solutions are presented in simple, analytical form so that their parametric dependence is apparent. To be sure, they are approximate solutions of the governing equations obtained by an integral method. The accuracy of the solutions can, of course, be improved by solving the equations exactly, but the present solutions seem adequate in providing a qualitative understanding of the physics provided by the simplified model.

It seems feasible to make some reasonable modifications to the present model to make it more realistic. For example, a more accurate model for calculating the convection heat transfer between the particle and the surrounding melt can be developed and used to account for the variation of the heat transfer rates around the particle surface. Also, phase changes of the particle during the course of its travel in the melt layer can be allowed by appropriate changes of the boundary conditions of the heat conduction equations. It would be interesting, and also important, to consider the dynamic effects of particles on ablation (for example, see Cheung et al.[6]), using the simple results presented in the present paper as a starting point.

Acknowledgment

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